especially that our formulas are generally valid only in the weak coupling case ($N\bar{V}\ll 1$), thus it may perhaps be improper to infer any definitive conclusions from experimental studies[7,8] on an anomalous (strong coupling) superconductor like lead.

Note added in proof. A recent paper by K. Maki, Progr. Theoret. Phys. (Kyoto) **29**, 603, 945 (1963), starts from the Green's function approach of Gorkov, and obtains results similar to ours.

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Bistatic Scattering from a Class of Lossy Dielectric Spheres with Surface Impedance Boundary Conditions*

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Expressions are derived for the bistatic scattering cross sections of spheres which exhibit sufficient electric and/or magnetic loss to permit each modal surface impedance in the Mie formulation to be replaced by a single impedance which is the intrinsic impedance of the lossy medium. Typical bistatic scattering curves are presented for several values of the characteristic impedance of the sphere medium.

IN a recent paper[1] by Wagner and Lynch, sufficient conditions are developed which effect zero electromagnetic backscatter from axially symmetric objects when illuminated along the axis of symmetry. The present paper considers the special case of scattering from spheres which exhibit sufficient loss to permit each modal surface impedance to be replaced by a single surface impedance which is the intrinsic impedance of the lossy medium. If this impedance is that of the ambient medium, the spheres have zero backscatter.

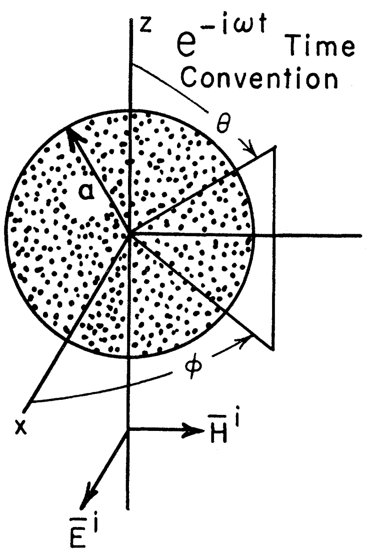

FIG. 1. The sphere geometry.

Referring to Fig. 1, the fields scattered by a sphere due to a plane wave incident from the $-z$ direction are,

$$E_\theta=\frac{-ie^{-i\omega t}e^{ik_2r}}{k_2r}\cos\phi\sum_{n=1}^{\infty}\frac{2n+1}{n(n+1)}\times\left[a_n\frac{P_n^1(\cos\theta)}{\sin\theta}+b_n\frac{\partial P_n^1(\cos\theta)}{\partial\theta}\right],\quad(1)$$

$$E_\phi=\frac{ie^{-i\omega t}e^{ik_2r}}{k_2r}\sin\phi\sum_{n=1}^{\infty}\frac{2n+1}{n(n+1)}\times\left[a_n\frac{\partial P_n^1(\cos\theta)}{\partial\theta}+b_n\frac{P_n^1(\cos\theta)}{\sin\theta}\right],\quad(2)$$

and the resulting normalized scattering cross sections are,

$$\frac{\sigma_E}{\pi a^2}=\lim_{r\to\infty}\left(\frac{2r}{a}\right)^2|E_\theta(\phi=0°)|^2,\quad(3)$$

$$\frac{\sigma_H}{\pi a^2}=\lim_{r\to\infty}\left(\frac{2r}{a}\right)^2|E_\phi(\phi=90°)|^2.\quad(4)$$

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[1] R. J. Wagner and P. J. Lynch, Phys. Rev. **131**, 21 (1963).

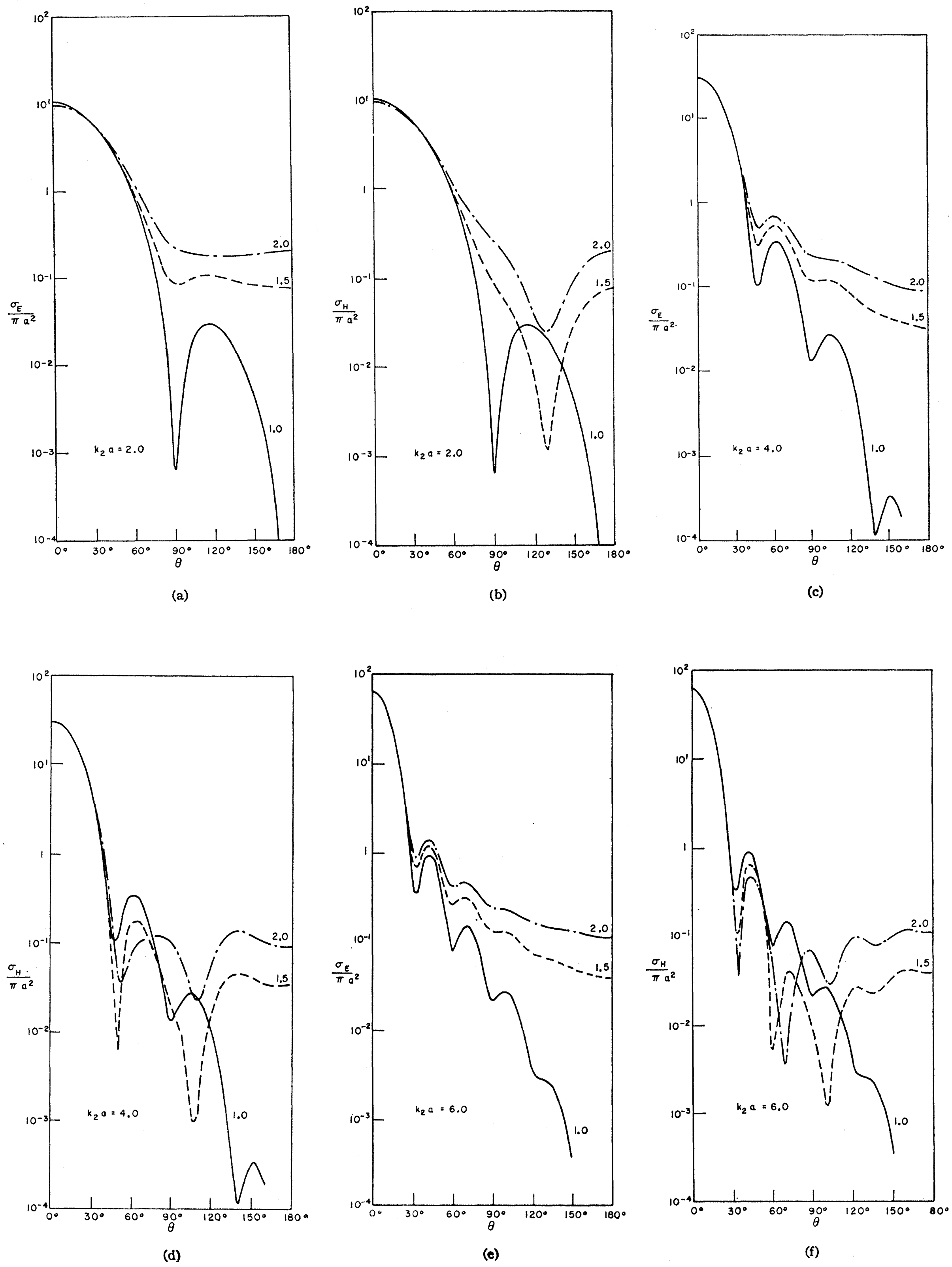

FIG. 2.—*Continued on next page.*

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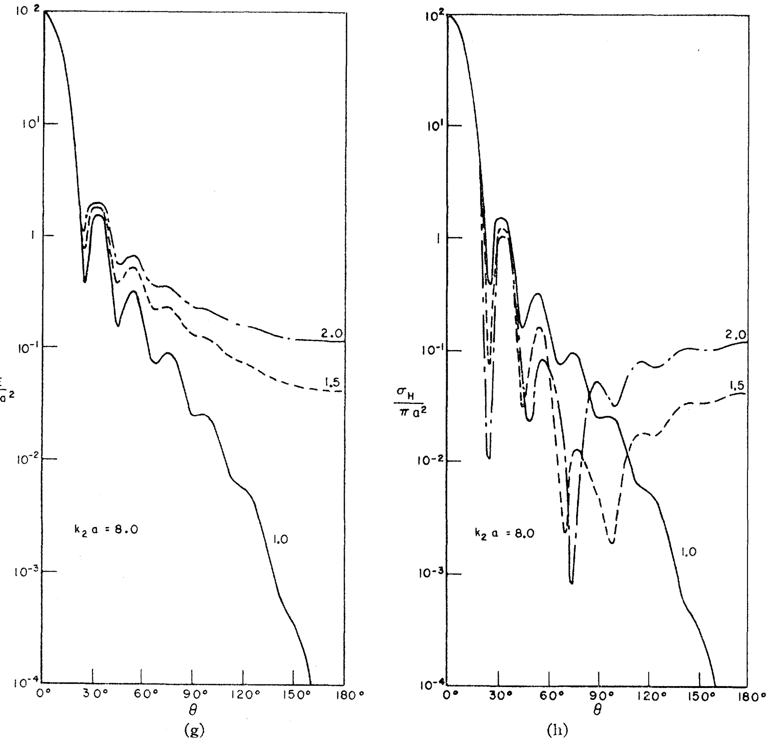

FIG. 2. E- and H-plane scattering patterns for spheres with $k_2a=2.0(2.0)$, 8.0 and $Z_1/Z_2=1.0$, 1.5, 2.0.

The scattering coefficients are given by[2]

$$a_n=-\frac{k_2aj_n(k_2a)-\left\{k_2a\dfrac{\mu_1}{\mu_2}\dfrac{j_n(k_1a)}{\hat{J}_n(k_1a)}\right\}\hat{J}_n(k_2a)}{k_2ah_n^{(1)}(k_2a)-\left\{k_2a\dfrac{\mu_1}{\mu_2}\dfrac{j_n(k_1a)}{\hat{J}_n(k_1a)}\right\}\hat{H}_n^{(1)}(k_2a)}, \tag{5}$$

$$b_n=-\frac{k_2aj_n(k_2a)-\left\{k_2a\dfrac{\epsilon_1}{\epsilon_2}\dfrac{j_n(k_1a)}{\hat{J}_n(k_1a)}\right\}\hat{J}_n(k_2a)}{k_2ah_n^{(1)}(k_2a)-\left\{k_2a\dfrac{\epsilon_1}{\epsilon_2}\dfrac{j_n(k_1a)}{\hat{J}_n(k_1a)}\right\}\hat{H}_n^{(1)}(k_2a)}, \tag{6}$$

where the symbol $\hat{J}_n(k_1a)=(\partial/\partial k_1r)[k_1rj_n(k_1r)]_{r=a}$, etc. The quantities enclosed in brackets are defined to be modal surface impedances (or admittances) normalized to the characteristic impedance of the ambient medium.[3] They are functions of the mode order, n; however, under the special conditions, $|k_1a|\gg1$, $|k_1a|\gg n$, $\mathrm{Im}[k_1a]\gg1$, they become essentially independent of mode order and may be approximated by the intrinsic impedance (or admittance) of the sphere medium normalized to that of the ambient medium. That is,

$$\left\{k_2a\frac{\mu_1}{\mu_2}\frac{j_n(k_1a)}{\hat{J}_n(k_1a)}\right\}\rightarrow i\frac{(\mu_1/\epsilon_1)^{1/2}}{(\mu_2/\epsilon_2)^{1/2}}=i\frac{Z_1}{Z_2}, \tag{7}$$

$$\left\{k_2a\frac{\epsilon_1}{\epsilon_2}\frac{j_n(k_1a)}{\hat{J}_n(k_1a)}\right\}\rightarrow i\frac{(\epsilon_1/\mu_1)^{1/2}}{(\epsilon_2/\mu_2)^{1/2}}=i\frac{Y_1}{Y_2}, \tag{8}$$

whence, the approximate scattering coefficients become,

$$a_n=-\frac{k_2aj_n(k_2a)-i\dfrac{Z_1}{Z_2}\hat{J}_n(k_2a)}{k_2ah_n^{(1)}(k_2a)-i\dfrac{Z_1}{Z_2}\hat{H}_n^{(1)}(k_2a)}, \tag{9}$$

$$b_n=-\frac{k_2aj_n(k_2a)-i\dfrac{Y_1}{Y_2}\hat{J}_n(k_2a)}{k_2ah_n^{(1)}(k_2a)-i\dfrac{Y_1}{Y_2}\hat{H}_n^{(1)}(k_2a)}. \tag{10}$$

Figure 2 presents the normalized scattering cross sections calculated from Eqs. (3), (4), (9), and (10) for spheres varying in sizes from $k_2a=2.0$ to $k_2a=8$ in steps of 2.0. The parameter Z_1/Z_2 was chosen to have the values 1, 1.5, and 2.0 and these are distinguished by the three types of lines in each figure. Note that since $\sigma_E(Z_1/Z_2)=\sigma_H(Z_2/Z_1)$ and $\sigma_H(Z_1/Z_2)=\sigma_E(Z_2/Z_1)$ these figures may also be used to obtain curves corresponding to $Z_1/Z_2=0.5$ and $Z_1/Z_2=0.667$. A more detailed discussion is available in Ref. 3.

[2] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, Inc., New York, 1941), p. 565.

[3] R. J. Garbacz, Antenna Laboratory, The Ohio State University Research Foundation, Report 925-5, 1961 (unpublished).